The Synthesis of Coupled Transmission Line All-Pass Networks in Cascades of 1 to n*

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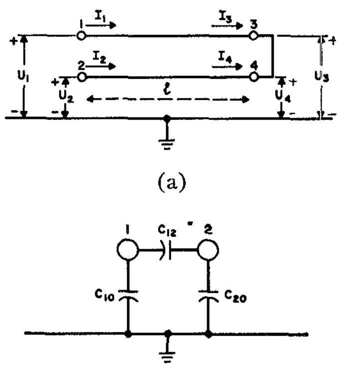
Summary—A homogeneous coupled line configuration realizing the characteristics of an all-pass network in the distributed network sense is useful for delay equalization in the UHF range. All-pass networks of first and second order are presented, while nth-order networks may be realized directly or built out of first- and second-order networks, analogous to the lumped-constant element network technique.



(a) (b)

Fig. 1.

I. Introduction

In the UHF range, the need for a delay-equalizing network has become evident, *e.g.*, for use in wide-band PCM transmission.

A distributed network consisting of coupled homogenous line sections is used here that allows precise allocation of the singularities in the complex plane and is the distributed equivalent to the lumped-constant all-pass network. All-pass networks (in the distributed sense) of first, second, or nth order are shown to be realizable in a very simple way. A delay equalization network of the nth order may be built by using either an nth-order section or combinations of lower-order sections, *e.g.*, $n/2$ second-order sections.

The basic structure consists of two symmetrical parallel conductors interconnected at one end and symmetrical between ground planes. A single unit of length $l=\lambda_{0/4}$ is a first-order section. A second-order section consists of a cascade of two sections of equal length and characteristic impedance, but with different coupling factors [1], [2].

In Section II of this paper, the equations of a pair of coupled lines will be considered and the appropriate boundary conditions introduced. In Section III, the transfer functions and delay functions are derived. In Section IV, the synthesis of delay networks consisting of first- and second-order sections will be discussed. Details regarding physical construction will be mentioned in Section V.

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II. Coupled Lines Between Ground Planes—General Considerations

The four-port network of Fig. 1(a) with terminals 3 and 4 disconnected can be represented by the following fourth order a matrix, [3], [4]:

$$\begin{bmatrix} U_1 \\ U_2 \\ I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & +j\dfrac{G_{22}}{\Delta}\sin\theta & -j\dfrac{G_{12}}{\Delta}\sin\theta \\ 0 & \cos\theta & -j\dfrac{G_{12}}{\Delta}\sin\theta & +j\dfrac{G_{11}}{\Delta}\sin\theta \\ +jG_{11}\sin\theta & +jG_{12}\sin\theta & \cos\theta & 0 \\ +jG_{12}\sin\theta & +jG_{22}\sin\theta & 0 & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} U_3 \\ U_4 \\ I_3 \\ I_4 \end{bmatrix}, \quad (1)$$

where

$$G_{11} = \frac{C_{10}+C_{12}}{\sqrt{\mu\epsilon}}; \qquad G_{22} = \frac{C_{20}+C_{12}}{\sqrt{\mu\epsilon}}$$

$$G_{12} = -\frac{C_{12}}{\sqrt{\mu\epsilon}}$$

$$\Delta = \begin{vmatrix} G_{11} & G_{12} \\ G_{12} & G_{22} \end{vmatrix} = \frac{1}{\mu\epsilon}\left[C_{10}C_{20} + C_{12}(C_{10}+C_{20})\right]. \quad (2)$$

C_{12}, C_{10} and C_{20} are the capacities per unit length between conductors [Fig. 1(b)]. $\theta=\beta l$ is the electrical length of the line. For the even and odd modes of propagation in the case of a symmetrical structure, the characteristic impedances between one conductor and ground are

$$Z_{oe} = \frac{\sqrt{\mu\epsilon}}{C_{10}} = \frac{\sqrt{\mu\epsilon}}{C_{20}}$$

$$Z_{oo} = \frac{\sqrt{\mu\epsilon}}{2\left[C_{12} + \frac{C_{10}C_{20}}{C_{10} + C_{20}}\right]}, \tag{3}$$

or since $C_{10} = C_{20}$

$$Z_{oo} = \frac{\sqrt{\mu\epsilon}}{C_{10} + 2C_{12}}.$$

Expressed in the susceptances per unit length

$$Z_{oe} = \frac{\sqrt{\mu\epsilon}}{C_{10}} = \frac{1}{G_{11} + G_{12}}$$

$$Z_{oo} = \frac{\sqrt{\mu\epsilon}}{C_{10} + 2C_{12}} = \frac{1}{G_{11} - G_{12}}. \tag{4}$$

The characteristic impedance of one conductor to ground, expressed in the characteristic impedances for odd and even modes of propagation is

$$Z_o = \sqrt{Z_{oe}Z_{oo}}$$
$$= \frac{\sqrt{\mu\epsilon}}{\sqrt{C_{10}C_{20} + C_{12}(C_{10} + C_{20})}}.$$
$$= \frac{1}{\sqrt{\Delta}}. \tag{5}$$

The ratio of even to odd mode impedance of the lines is expressed by

$$\rho = \frac{Z_{oe}}{Z_{oo}} = \frac{G_{11} - G_{12}}{G_{11} + G_{12}}. \tag{6}$$

The coupling factor is usually defined as

$$k = \frac{\rho - 1}{\rho + 1} = \frac{Z_{oe} - Z_{oo}}{Z_{oe} + Z_{oo}} = -\frac{G_{12}}{G_{11}}.$$

The following analysis will assume:

1) The lines are lossless.
2) Each pair of lines is symmetrical; $C_{10} = C_{20}$ (or $G_{11} = G_{22}$).
3) The electrical length of cascaded sections is equal; $\theta_1 = \theta_2 = \cdots = \theta_n$.
4) The characteristic impedance of cascaded sections is equal; $Z_{01} = Z_{02} = \cdots = Z_{0n}$.
5) The coupling factor varies; $\rho_1 \neq \rho_2 \neq \cdots \rho_n$.

For cascaded sections, the a matrix may be formed by multiplication of the a matrices of the individual sections (Appendix I). The boundary conditions

$$U_3 = U_4$$
$$I_3 = -I_4 \tag{7}$$

are then introduced allowing the derivation of a second-order y matrix from the fourth-order a matrix. The resulting transfer functions for first-, second- and nth-order all-pass sections are derived in Appendix I.

III. Coupled Line All-Pass Sections of Order 1-n

In the following paragraphs the coordinates of the singularities in the complex plane and the delay vs frequency function will be derived from the transfer function.

The First Order Section

The transfer function is [see Appendix I, (31)]

$$\frac{U_2}{U_1}(\theta) = \frac{\sqrt{\rho} - j\tan\theta}{\sqrt{\rho} + j\tan\theta} \tag{8}$$

with

$$j\theta = j\frac{\pi}{2}\frac{\omega}{\omega_0}$$

for a quarter-wavelength section. Introducing the complex variable

$$s = \sigma + j\theta = \frac{\pi}{2\omega_0}(\alpha + j\omega) \tag{9}$$

in (8) gives

$$\frac{U_2}{U_1}(s) = \frac{\sqrt{\rho} - \tanh s}{\sqrt{\rho} + \tanh s}. \tag{10}$$

The singularities occur at $\tanh s = \pm\sqrt{\rho}$. (11)
In general, the solution of

$$\tanh(\sigma + j\theta) = x + jy$$

is

$$\sigma = \frac{1}{4}\ln\frac{(1 + x)^2 + y^2}{(1 - x)^2 + y^2}$$

$$\theta = \frac{1}{2}\tan^{-1}\frac{2y}{1 - x^2 - y^2} \pm k\pi. \tag{12}$$

Here we obtain

$$\theta = \frac{1}{2}\tan^{-1}[o] + k\pi$$

$$\sigma = \pm\frac{1}{2}\ln\frac{\sqrt{\rho} + 1}{\sqrt{\rho} - 1}. \tag{13}$$

Or, expressed in α and ω:

$$\frac{\omega}{\omega_0} = 1 \pm 2k$$

$$\frac{\alpha}{\omega_0} = \pm\frac{1}{\pi}\ln\frac{\sqrt{\rho} + 1}{\sqrt{\rho} - 1}. \tag{14}$$

In (13) and (14) the + sign in the expressions for σ and α applies to the zeros of transmission; the − sign to the natural modes.

The singularities in the s plane are periodic in ω_0 (Fig. 2), and for a constant ω_0 (a constant length of the section) can vary in σ only, which corresponds to a variation in peak height accomplished by varying the coupling between a pair of lines.

The delay vs frequency function is

$$\tau(\omega) = \frac{d}{d\omega}\left[2\tan^{-1}\frac{\tan\frac{\pi}{2}\frac{\omega}{\omega_0}}{\sqrt{\rho}}\right]$$

$$= \frac{\pi}{\omega_0}\;\frac{\sqrt{\rho}}{\rho\cos^2\frac{\pi}{2}\frac{\omega}{\omega_0} + \sin^2\frac{\pi}{2}\frac{\omega}{\omega_0}}. \qquad (15)$$

For a few typical values of ρ, the delay as a function of frequency is shown in Fig. 3.

When ω_0 and α are given, the factor ρ may be derived from (14).

$$\rho = \left[\frac{e^{\pi\alpha/\omega_0} + 1}{e^{\pi\alpha/\omega_0} - 1}\right]^2. \qquad (16)$$

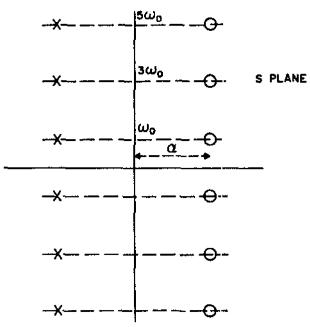

Fig. 2.

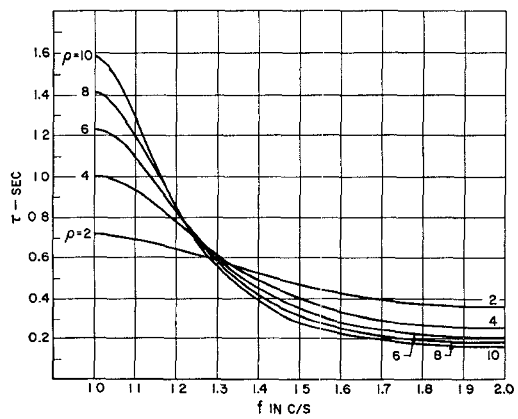

Fig. 3.

As an example, a few values of α/ω_0 and ρ are tabulated in the following:

α/ω_0	ρ
1.19	1.1
0.984	1.2
0.730	1.5
0.561	2.0
0.350	4.0
0.205	10.0.

It is possible to combine a number of first-order sections of different length (different ω_0) to approximate a delay function in an interval, *e.g.*, $\omega/\omega_0 = 1$ to $\omega/\omega_0 = 2$ (ω_0 applies to first section only). The resulting curve will have a period determined by the least common multiple of the ω_0's used.

The use of second-order sections allows for variation of delay peak height and position for each section within certain limits. For a number of sections of equal length (equal ω_0) a periodic delay vs frequency function, usually preferred in distributed constant network synthesis, is thus realized.

The Second Order Section

The transfer function (35)

$$\frac{U_2}{U_1}(\theta) = \frac{1 - r\tan^2\theta - jR\tan\theta}{1 - r\tan^2\theta + jR\tan\theta} \qquad (17)$$

where

$$r = \frac{Z_{001}}{Z_{002}} = \frac{\sqrt{\rho_2}}{\sqrt{\rho_1}} \quad \text{and} \quad R = \frac{1}{\sqrt{\rho_1}} + \frac{1}{\sqrt{o}}.$$

Substitution of the complex variable

$$s = \sigma + j\theta = \frac{\pi}{2\omega_0}(\alpha + j\omega)$$

results in

$$\frac{U_2}{U_1}(s) = \frac{1 + r\tanh^2 s - R\tanh s}{1 + r\tanh^2 s + R\tanh s}. \qquad (18)$$

The zeros of transmission occur at

$$\tanh s = \frac{R}{2r} \pm \frac{j}{2r}\sqrt{4r - R^2}. \qquad (19)$$

The natural modes occur at

$$\tanh s = -\frac{R}{2r} \pm \frac{j}{2r}\sqrt{4r - R^2}. \qquad (20)$$

The right-hand side of (19) and (20) is complex for

$$4r - R^2 > 0. \qquad (21)$$

Eq. (21) does not limit the choice of ρ_1 and ρ_2 greatly as is shown in Appendix II. Substitution of

$$x = \pm \frac{R}{2r}$$

$$y = \pm \frac{1}{2r} \sqrt{4r - R^2}$$

in (12) gives the coordinates of the following singularities:

$$\sigma = \pm \frac{1}{4} \ln \frac{r+1+R}{r+1-R}$$

$$\theta = \frac{1}{2} \tan^{-1} \frac{\sqrt{4r - R^2}}{r-1} \pm k\pi \tag{22}$$

or

$$\frac{\alpha}{\omega_0} = \pm \frac{1}{2\pi} \ln \frac{r+1+R}{r+1-R}$$

$$\frac{\omega}{\omega_0} = \frac{1}{\pi} \tan^{-1} \frac{\sqrt{4r - R^2}}{r-1} \pm 2k. \tag{23}$$

In (22) and (23), the + sign for σ and α applies to the zeros of transmission; the − sign to the natural modes. The singularities in the s plane (Fig. 4) are periodic in ω_0.

$$\omega_p = \omega_z = k\omega_0 \pm \delta \quad \text{with} \quad k = 1, 3, 5, \cdots.$$

It is possible to vary δ for a constant ω_0. Thus, the peak position and the peak height may be varied for a constant ω_0, that is, a constant length of the section.

The delay function

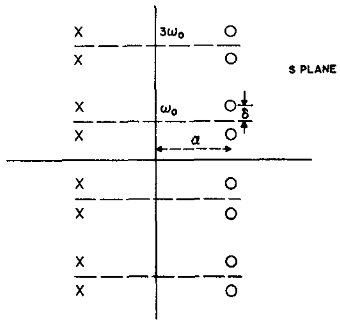

Fig. 4.

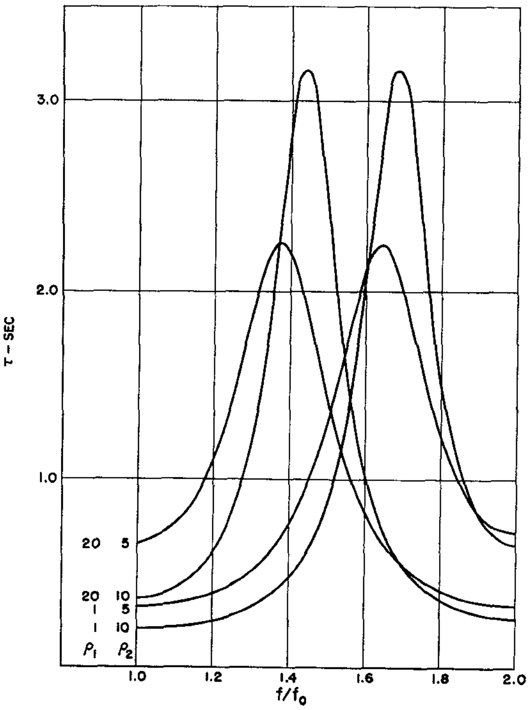

Fig. 5.

$$r(\omega) = \frac{d}{d\omega}\left[2 \tan^{-1} \frac{R \tan \frac{\pi}{2}\frac{\omega}{\omega_0}}{1 - r \tan^2 \frac{\pi}{2}\frac{\omega}{\omega_0}}\right] = R \cdot \frac{\pi}{\omega_0} \frac{\cos^2 \frac{\pi}{2}\frac{\omega}{\omega_0} + r \sin^2 \frac{\pi}{2}\frac{\omega}{\omega_0}}{\cos^4 \frac{\pi}{2}\frac{\omega}{\omega_0} + (R^2 - 2r) \sin^2 \frac{\pi}{2}\frac{\omega}{\omega_0} \cos^2 \frac{\pi}{2}\frac{\omega}{\omega_0} + r^2 \sin^4 \frac{\pi}{2}\frac{\omega}{\omega_0}}. \tag{24}$$

For typical values of ρ_1 an d ρ_2, the delay as a function of frequency is shown in Fig. 5.

To determine ρ_1 and ρ_2 for given singularities (22) has to be solved. Substitution of

$$r = \frac{\sqrt{\rho_2}}{\sqrt{\rho_1}} \quad \text{and} \quad R = \frac{1}{\sqrt{\rho_1}} + \frac{1}{\sqrt{\rho_2}}$$

in (22) gives

$$\tanh \sigma = \sqrt{\rho_2} - \sqrt{\rho_2 - 1}$$

$$\tan \theta = \frac{\sqrt{\rho_2}(\sqrt{\rho_1} - \sqrt{\rho_2}) + (\sqrt{\rho_2 - 1})(\sqrt{\rho_1} + \sqrt{\rho_2})}{\sqrt{(4\rho_2 - 2)\sqrt{\rho_1\rho_2} - (\rho_1 + \rho_2)}}. \tag{25}$$

Eq. (25) may be solved for ρ_2:

$$\rho_2 = \coth^2 (2\sigma). \tag{26}$$

With ρ_2, σ and θ known, ρ_1 may then be found by solving the quadratic equation in $\sqrt{\rho_1}$:

$$\rho_1[\tan^2\theta + (2\rho_2 - 1) + 2\sqrt{\rho_2(\rho_2 - 1)}] - \sqrt{\rho_1}\cdot 2\sqrt{\rho_2}[(2\rho_2 - 1)\tan^2\theta + 1] + \rho_2[\tan^2\theta + 2\rho_2 - 1 - 2\sqrt{\rho_2(\rho_2 - 1)}] = 0. \quad (27)$$

It is thus possible to determine the physical constants of the second-order sections from a given plot of singularities and to synthesize a delay network consisting of a number of these second-order sections in sequence.

For the examples shown in Fig. 5, the values of σ and θ (or α/ω_0 and ω/ω_0) with the corresponding values of ρ_1 and ρ_2 are shown in the following table:

σ	θ	α/ω_0	ω/ω_0	ρ_1	ρ_2
0.1637	± 0.4957	0.1042	1 ± 0.6844	1	10
0.2406	± 0.5647	0.1532	1 ± 0.6405	1	5
0.1637	± 0.8763	0.1042	1 ± 0.4421	20	10
0.2406	± 0.9764	0.1532	1 ± 0.3784	20	5

The n*th-Order Section*

Eq. (36) indicates the expressions for the matrix elements of a cascade of n sections. The transfer function may be formed, and the delay function derived from this in very much the same manner as was done for the first and second order. The effort involved in each application will have to be weighed against the possibility of using combinations of first- and second-order sections to achieve the same result.

The limit of the nth-order system for $n \to \infty$ and $l \to 0$, keeping the total length constant, leads to the coupled inhomogeneous line. The equations of the nth-order section will be instrumental in obtaining these, as is done for the directional coupler by Fel'dshtein [4], [5]. Due to the different (nonperiodic) frequency characteristic of the transfer function of such a line, the present study will not include it.

IV. The Synthesis of Delay Networks from Given Singularities in the Complex Plane

When the approximation problem has been solved, preferably realizing a periodic singularity pattern, the realization of the delay network (following the familiar method used for lumped-constant all-pass networks) only requires identification of each singularity in a horizontal strip of width ω_0 in the complex plane with a first- or second-order all-pass section, and computation of the coupling factor values according to the method of Section III. It may be necessary to readjust the singularity pattern if values of ρ are obtained which are not realizable with known techniques, *e.g.*, in coupled line structures using broadside coupled striplines.

The coupled line sections need to be interconnected either vertically or in the horizontal plane. The characteristic impedance of the interconnecting stubs should closely approximate the characteristic impedance of the sections. Their length should be kept as short as possible, since this contributes to the constant delay of the network.

Distance between the horizontally-separated sections should be of the same magnitude as the ground plates separation to avoid coupling between sections.

V. Construction of the Coupled Line Sections

For construction in stripline, between groundplates, detailed information is available in literature.

For a moderate degree of coupling, the coplanar structure is used [6], [Fig. 6(a)]. The broadside-coupled stripline pair permits a higher degree of coupling [7], [8], [Fig. 6(b) and (c)].

It is thus possible to relate given characteristic impedance and coupling factor with the dimensions of the stripline pair. The finite thickness of the conductors may be corrected for, if necessary [9].

The characteristic impedance of the stubs interconnecting the sections should come close to the characteristic impedance of the coupled pair. Interconnection of the ground planes is necessary near the short at the end of each section.

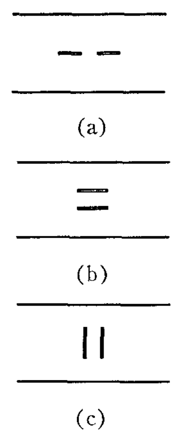

Fig. 6.

VI. Conclusion

For delay function approximation, the distributed all-pass section may be used in the same manner as the lumped-constant section and makes possible accurate network synthesis from singularity plots in the complex plane.

Appendix I

The a matrix of (1) is of the following form:

$$\begin{bmatrix} A & B & C & D \\ B & A & D & C \\ \Delta C & -\Delta D & A & -B \\ -\Delta D & \Delta C & -B & A \end{bmatrix}. \quad (28)$$

Multiplication of two or more of these matrices, for two or more sections in cascade, when the condition 1–5 of Section II apply, results in matrices of the same type with only the elements of the first row and Δ as parameters.

The transfer function derived here will apply to the general case of 1-n sections in cascade. The boundary conditions $U_3 = U_4$ and $I_3 = -I_4$ substituted in (28) result in the two port Y matrix equation:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{1}{2(A+B)(C-D)} \begin{bmatrix} \Delta(C-D)^2 + (A+B)^2 & \Delta(C-D)^2 - (A+B)^2 \\ \Delta(C-D)^2 - (A+B)^2 & \Delta(C-D)^2 + (A+B)^2 \end{bmatrix} \cdot \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}. \quad (29)$$

The transfer function, with port 2 terminated in $Y_o = \sqrt{\Delta}$:

$$\frac{U_2}{U_1} = -\frac{Y_{21}}{Y_{22}+Y_0} = \frac{(A+B) - \sqrt{\Delta}(C-D)}{(A+B) + \sqrt{\Delta}(C-D)}. \quad (30)$$

Eq. (30) can be used to derive the transfer function for all-pass sections of any order.

The first-order section: substitution of the matrix elements of (1) in (30) gives

$$\frac{U_2}{U_1}(\theta) = \frac{\cos\theta - \sqrt{\Delta} j \left(\frac{G_{12}+G_{22}}{\Delta}\right)\sin\theta}{\cos\theta + \sqrt{\Delta} j \left(\frac{G_{12}+G_{22}}{\Delta}\right)\sin\theta},$$

or, applying (4)–(6),

$$\frac{U_2}{U_1}(\theta) = \frac{\sqrt{\rho} - j\tan\theta}{\sqrt{\rho} + j\tan\theta}. \quad (31)$$

The second-order section: multiplication of two matrices as shown in (1) results in the following expressions for the elements of the over-all 4×4 matrix:

$$A = \cos^2\theta + \frac{G_2'G_1' - G_2G_1}{\Delta}\sin^2\theta$$

$$B = \frac{G_2'G_1 - G_2G_1'}{\Delta}\sin^2\theta$$

$$C = +j\frac{G_1+G_2}{\Delta}\sin\theta\cos\theta$$

$$D = -j\frac{G_1'+G_2'}{\Delta}\sin\theta\cos\theta \quad (32)$$

where

$G_i' = (G_{12})_i$ for $i = 1, 2$

$G_i = (G_{11})_i$ for $i = 1, 2$

i = Section Number [see Fig. 7(a)]

$\Delta = \Delta_1 = \Delta_2$; the characteristic impedance is constant.

Substitution of (32) in (30) gives the transfer function

$$\frac{U_2}{U_1}(\theta) = \frac{\cos^2\theta + \frac{G_2'G_1' + G_2'G_1 - G_2G_1 - G_2G_1'}{\Delta}\sin^2\theta - j\frac{G_1+G_2+G_1'+G_2'}{\sqrt{\Delta}}\sin\theta\cos\theta}{\cos^2\theta + \frac{G_2'G_1' + G_2'G_1 - G_2G_1 - G_2G_1'}{\Delta}\sin^2\theta + j\frac{G_1+G_2+G_1'+G_2'}{\sqrt{\Delta}}\sin\theta\cos\theta}.$$

From (4)–(6), we find

$$\frac{G_2'G_1' + G_2'G_1 - G_2G_1 - G_2G_1'}{\Delta} = -\frac{(G_{11}-G_{12})_2}{(G_{11}-G_{12})_1} = -\frac{Z_{oo1}}{Z_{oo2}} = -r \quad (33)$$

$$\frac{G_1+G_2+G_1'+G_2'}{\sqrt{\Delta}} = \sqrt{\left(\frac{G_{11}+G_{12}}{G_{11}-G_{12}}\right)_1} + \sqrt{\left(\frac{G_{11}+G_{12}}{G_{11}-G_{12}}\right)_2} = \frac{1}{\sqrt{\rho_1}} + \frac{1}{\sqrt{\rho_2}} = R. \quad (34)$$

The transfer function becomes

$$\frac{U_2}{U_1}(\theta) = \frac{[1 - r\tan^2\theta] - jR\tan\theta}{[1 - r\tan^2\theta] + jR\tan\theta}. \quad (35)$$

The nth-order section: by repeated matrix multiplications the matrix elements for n sections in cascade, numbered as in Fig. 7(b), are obtained in the following:

$$A_n = a_nA_{n-1} + b_nB_{n-1} + \Delta c_nC_{n-1} - \Delta d_nD_{n-1}$$

$$B_n = a_nB_{n-1} + b_nA_{n-1} - \Delta c_nD_{n-1} + \Delta d_nC_{n-1}$$

$$C_n = a_nC_{n-1} + b_nD_{n-1} + c_nA_{n-1} - d_nB_{n-1}$$

$$D_n = a_nD_{n-1} + b_nC_{n-1} - c_nB_{n-1} + d_nA_{n-1}. \quad (36)$$

In (36) the matrix elements of multiple sections are represented by capital letters, those of single sections by small letters. It is thus possible to form expressions for

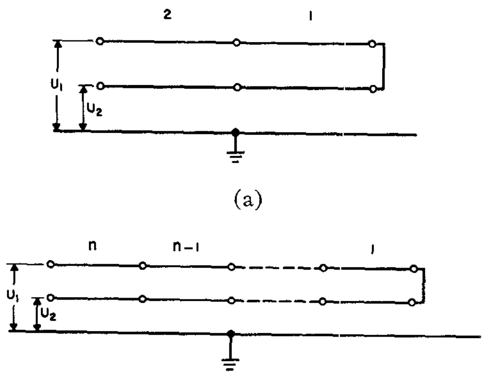

Fig. 7.

(A_n+B_n), (C_n-D_n), and U_2/U_1 for any number of sections in cascade.

Appendix II

The roots of numerator and denominator of the second-order transfer function are complex if (21) is satisfied.

$$4r - R^2 > 0$$

or

$$4\sqrt{\frac{\rho_2}{\rho_1}} - \left(\frac{1}{\sqrt{\rho_1}} + \frac{1}{\sqrt{\rho_2}}\right)^2 > 0. \qquad (37)$$

Eq. (37) will limit the choice of ρ_1 and ρ_2 somewhat, but not severely.

When $\rho_2 > \rho_1$ (37) is always satisfied for ρ_1 and $\rho_2 \geq 1$. When $\rho_1 > \rho_2$, the following table indicates the limitation on the choice of the parameters:

ρ_1	ρ_2
1	1
<3.81	1.1
<6.79	1.2
<12.7	1.3
<20.8	1.5
<67.9	2.0.

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